Chapter 11
Vibratory Motion Chapter Review

EQUATIONS:

- $\boldsymbol{F}_{spring} = kx(-\boldsymbol{i})$ [This expression yields the magnitude and directional nature of the force a spring applies to a mass when the spring has been compressed or elongated a distance *x* units from equilibrium. The negative sign denotes the fact that the force is always toward the equilibrium position (when the mass is on the negative side of equilibrium, *x* is negative and the force is calculated correctly as being in the positive direction). The variable *k* is defined as the *spring constant*. Technically, it tells you how many newtons it takes to compress or elongate the spring one meter (its units are nts/m). Qualitatively, it tells you how rigid the spring is.]

- $T = \frac{1}{\nu}$ [This is an expression that is ALWAYS TRUE. It states that the period *T* of an oscillatory system is the inverse of the frequency ν. Note that the units for the period are *seconds/cycle*, whereas the units for the frequency are *cycles/second*.]

- $\omega = 2\pi\nu$ [This is an expression that is ALWAYS TRUE. It states that the angular frequency ω of a vibrating body is equal to 2π times the frequency ν of the system. The units of angular frequency are *radians/second*, which means the units for the 2π quantity must be *radians/cycle*.]

- $\omega_{spring} = \left(\frac{k}{m}\right)^{\frac{1}{2}}$ [True only for an ideal spring system (i.e., one that oscillates with simple harmonic motion), this is the relationship between the angular frequency ω of an oscillating mass *m* attached to a spring of spring constant *k*.]

- $E_{tot} = \frac{1}{2}kA^2$ [This is the total energy of a spring system oscillating in simple harmonic motion, where *k* is the spring constant and *A* is the amplitude of the motion.]

- $v_{max} = \omega A$ [This is the magnitude of the maximum velocity of any oscillating system, where ω is the angular frequency of the system and *A* is the system's amplitude. The maximum velocity occurs at equilibrium where the accelerating force is completely exhausted.]

- $a_{max} = \omega^2 A$ [This is the magnitude of the maximum acceleration of any oscillating system, where ω is the angular frequency of the system and *A* the system's amplitude. The maximum acceleration occurs at the extremes where the accelerating force is the greatest.]

- $(acceleration\ term) + (constant)(position\ term) = 0$ [This is the characteristic equation for simple harmonic motion. If you can manipulate a Newton's Second Law expression into the form $a + (K)x = 0$, or $\alpha + (K)\theta = 0$, where K is some constant (i.e., k/m if the system is an ideal spring, etc.), previous observations and derivations allow us to know that the motion IS simple harmonic in nature, and that the angular frequency of the motion is $\omega = \sqrt{K}$.]

- $x(t) = A\,sin(\omega t + \phi)$ [This is the equation that characterizes the position of an oscillating body as a function of time (example: 3 sin (12t +.7) . . .). Note that A is the amplitude of the motion, ω is the angular frequency, and ϕ is the phase shift (the inclusion of ϕ allows you to start the clock whenever you want within the oscillatory cycle).]

- $\omega_{simple\ pendulum} = \left(\frac{g}{L}\right)^{\frac{1}{2}}$ [True only for a *simple* pendulum system (i.e., a massless string with a point-mass bob at one end) moving in *small amplitude oscillation*, this is the relationship between the system's angular frequency ω, its pendulum length L, and the acceleration of gravity g. Note that this expression suggests a clever way to determine the acceleration of gravity on the moon or some other celestial body.]

<u>COMMENTS, HINTS, and THINGS to be aware of</u>:

- The temptation is to memorize formulas. **Know the relationships**, **BUT** beyond all else, understand the *reasoning* behind the relationships. I remember an AP test problem that assumed a non-linear spring (example: $F = -kx^5$) and asked qualitative questions from there. An example: *If you elongate the spring a distance L and release it, the period is T. What happens to T if the spring is released at 1.5L?* The moral? Understand the concepts. Understand why the relationships are as they are. Don't memorize relationships mindlessly.

- If you are given an exotic oscillating system and asked to **determine** the **system's period**, **the trick is**:
  --Show that the motion is *simple harmonic motion* by writing out N.S.L., then put that equation into the form *(acceleration)* + *(constant)(position)* = *0* . . . (this is the characteristic equation for simple harmonic motion.)
  --The *constant* will equal the square of the motion's angular frequency.
  --You know the relationship between *angular frequency*, *frequency*, and *period*. Use that relationship to derive your *period* expression.

- A **spring constant tells you**:
  --How rigid the spring is.
  --How much force per unit distance is required to elongate or compress the spring.

- To **determine** a **spring constant**:
  --You need to determine the amount of force required to elongate or compress the spring some given distance.
  --You could either be given that information straight away, or you could be told the distance d a given amount of mass m displaces the spring (remember, the force that is being applied in such a case is that of gravity--it is $f_{grav} = mg$) when hung vertically and allowed to slowly lower to its new equilibrium position.